The energy dependence of the Pomeron and the eikonalized Pomeron

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Abstract

The one-to-one connection between the eikonal phase and the ratio of the elastic and total cross section is shown. Based on new experimental data of Collaboration CDF we analyzed intercept and power of the logarithmic growth of the Born and total Pomeron amplitude.

Now we have a long and plenty discussion of the energy dependence of the elastic and total cross section in the hadron - hadron scattering [1, 2]. Some models with the different hypothesis (see for example [3]. lead to

$$\sigma_{tot} \sim lns.$$

One of the recent analysis of the plenty experimental material has been made in [4]. The conclusion was that this analysis gives strong evidence of a $\log(s/s_0)$ dependence at current energies rather than of $\log^2(s/s_0)$ and demonstrates that the odderon is not necessary to explain experimental data. But many models (for example [5]) were based on the idea of "supercritical" Pomeron exchange, with $\varepsilon = \alpha(0) - 1 > 0$, which after unitarization results in the Froissart saturation of the total cross sections

$$\sigma_{tot} \sim ln^2 s.$$

In [6] the overall analysis of experimental material was made ¿from viewpoint of the soft and hard supercritical Pomeron where the $\log^2(s/s_0)$ was obtained. Good agreement with data on deep inelastic scattering and photoproduction data is reached in the case when the non-perturbative component of the Pomeron is governed by the "maximal" behaviour, i.e. like $\ln^2(s)$ [7].

One can make different remarks on this and others analyses and on the values of the experimental material used. Essential uncertainty in the values of σ_{tot} was shown in [8]. Now we have the large discussion about of the value of σ_{tot} at $\sqrt{s} = 1.8 TeV$. The last work of the CDF Collaboration [9] gives

$$(1+\rho^2)\sigma_{tot} = 62.64 \pm 0.95 (mb) \quad at \quad \sqrt{s} = 546 GeV,$$

$$(1+\rho^2)\sigma_{tot} = 81.83 \pm 2.29 (mb) \quad at \quad \sqrt{s} = 1.8 TeV,$$

and

$$\delta(s_1) = \sigma_{elast}/\sigma_{tot} = 0.210 \pm .002 \quad at \quad \sqrt{s} = 546 GeV,$$

$$\delta(s_2) = \sigma_{elast}/\sigma_{tot} = 0.246 \pm .004 \quad at \quad \sqrt{s} = 1.8 TeV.$$

Let is compare this value of σ_{tot} at $\sqrt{s} = 1.8 TeV$. with the previous value which was equals to $72 mb$ The lasts two relations have small errors as a consequence of the cancellation of some errors. As we will show further, the relation of these two values is more interesting and helps us to obtain the intercept of the Born and eikonalized Pomeron. Let us denote the ratio of these two last values by $\Delta(s_{12})$

$$\Delta(s_{12}) = \frac{\delta(s_1)}{\delta(s_2)} = \frac{\sigma_{el}(s_1)\cdot\sigma_{tot}(s_2)}{\sigma_{el}(s_2)\cdot\sigma_{tot}(s_1)}.$$

Now we consider the ordinary representation for the Born term of the scattering amplitude at $\sqrt{s} \geq 540 GeV$. At such large energies, we can neglect the contribution of the non-leading Regge terms, and in this analysis, for the simplicity, we neglect the real part of the scattering amplitude

$$T(s,t) = i h s^{\alpha(t)-1} e^{R_0^2 \cdot t/2} \tag{1}$$

with the linear trajectories $\alpha(t) = \alpha(0) + \alpha_1 t$ and $\alpha(0) = 1 + \varepsilon$. The differential elastic, total elastic and total cross section have the following forms:

$$\frac{d\sigma}{dt} = \pi \mid T(s,t) \mid; \quad \sigma_{el} = \int_{-\infty}^{0} \frac{d\sigma}{dt} dt; \quad \sigma_{tot} = 4\pi Im T(s,0).$$

For the amplitude (1) we obtain

$$\sigma_{el} = \pi h^2 \int_{-\infty}^{0} s^{2\cdot(\alpha(t)-1)} e^{R_0^2 \cdot t} dt \quad = \quad 2\pi \frac{h^2 s^{2\varepsilon}}{R^2} \tag{2}$$

and

$$\sigma_{tot} = 4\pi h s^{\varepsilon} \tag{3}$$

where $R^2 = R_0^2 \cdot (1 + \alpha \cdot lns)$.

Hence, the relation σ_{el}/σ_{tot} is

$$\frac{\sigma_{el}}{\sigma_{tot}} = \delta(s) = \frac{h}{2R^2} \cdot s^{\varepsilon}. \tag{4}$$

Using the value of $\Delta(s_{12})$ we can find the intercept of the Pomeron

$$\begin{aligned} \varepsilon = & \frac{ln(\Delta(s_{12}))}{ln(s_1/s_2)} + ln[\frac{1+\alpha ln(s_1)}{1+\alpha ln(s_2)}]/ln(s_1/s_2) \\ & = \varepsilon_0 + \varepsilon_1. \end{aligned} \tag{5}$$

If we take the amplitude in the form

$$T(s,t) = ih \cdot ln^n(s) \cdot e^{R_0^2 \cdot t/2}, \tag{6}$$

we can calculate the value of n.

$$\begin{aligned} n = & \frac{ln(\Delta(s_{12}))}{ln(ln(s_1)/ln(s_2))} + ln[\frac{1+\alpha ln(s_1)}{1+\alpha ln(s_2)}]/ln(ln(s_1)/ln(s_2)). \\ & = n_0 + n_1 \end{aligned} \tag{7}$$

Now let us consider the total eikonal amplitude

$$T(s,t) = i \int \rho d\rho J_0(\rho\Delta)(1 - e^{i\chi(s,\rho)}), \tag{8}$$

where

$$i\chi(s,\rho) = i \int \Delta J_0(\rho\Delta) T^B(s, -\Delta^2) d\Delta. \tag{9}$$

For the eikonal phase we derive

$$i\chi(s,\rho) = -\frac{hs^{\varepsilon^B}}{R^2} e^{-\rho^2/(2R^2)} = -X \cdot e^{-\rho^2/(2R^2)}. \tag{10}$$

It can be shown that the total and elastic cross section can be represented as

$$\sigma_{tot} = R^2 \sum_{k=1}^{\infty} (-1) \frac{(-X)^k}{k!k}, \tag{11}$$

$$\sigma_{el} = R^2 \ \{2 \sum_{k=1}^{\infty} (-1) \frac{(-X)^k}{k!k} - \sum_{k=1}^{\infty} (-1) \frac{(-2X)^k}{k!k}\}. \tag{12}$$

Hence, its ratio depends only on X. The calculation confirms this and, for the experimental values we have the one-to-one correspondence with X_i.

$$\delta(546) = .210 \leftrightarrow X(546) = 1.38$$

$$\delta(1800) = .246 \leftrightarrow X(1800) = 1.862$$

Based on these values X_i, we can obtain the intercept ε^B or a power of the logarithmic growth n^B. They have nearly the same form as (5),(7)

$$\begin{aligned} \varepsilon^B = \frac{ln[X(s_1)/X(s_2)]}{ln(s_1/s_2)} + ln[\frac{1+\alpha ln(s_1)}{1+\alpha ln(s_2)}]/ln(s_1/s_2) \\ = \varepsilon_0^B + \varepsilon_1; \end{aligned} \tag{13}$$

$$\begin{aligned} n^B = \frac{ln[X(s_1)/X(s_2)]}{ln(ln(s_1)/ln(s_2))} + ln[\frac{1+\alpha ln(s_1)}{1+\alpha ln(s_2)}]/ln(ln(s_1)/ln(s_2)) \\ = n_0^B + n_1. \end{aligned} \tag{14}$$

Using the recent experimental data we can calculate

$$\varepsilon_0 = 0.066; \quad \varepsilon_0^B = 0.126.$$

$$n_0 = 0.913 \quad n_0^B = 1.73.$$

The values of ε_1 and n_1 weakly depend on the value of α. The values of the total cross sections heavily depend on these values of α.

So, for $R^2 = R_0^2(1+\alpha ln(s))$ we have

for $\alpha = 0.06$

$R_0^2 =$	6.8	6.9	7.0	7.1	
$\sigma_{tot}(546) =$	59.5	60.37	61.24	62.12	
$\sigma_{tot}(1800) =$	79.44	80.6	81.8	82.94	

for $\alpha = 0.05$

$R_0^2 =$	7.3	7.4	7.5	7.6	7.7
$\sigma_{tot}(546) =$	59.3	60.1	60.91	61.72	62.54
$\sigma_{tot}(1800) =$	78.55	79.6	80.7	81.8	82.85

for $\alpha = 0.04$

$R_0^2 =$	7.8	8.0	8.2
$\sigma_{tot}(546) =$	58.45	59.9	61.45
$\sigma_{tot}(1800) =$	76.74	78.7	80.7

Let us remember that the experimental values of σ_{tot} are

$(1+\rho^2)\sigma_{tot}(546.) = 62.64(mb)$ CDF

$(1+\rho^2)\sigma_{tot}(546.) = 63.5(mb)$ UA4

$(1+\rho^2)\sigma_{tot}(1800) = 81.83(mb)$ CDF.

It is clear from these Tables and Fig.1 that the value of α cannot exceed 0.05, otherwise we will get the wide divergence from the experimental data of the σ_{tot}. Moreover, we can conclude that the experimental value of σ_{tot} at $\sqrt{s} = 1800GeV$ cannot be less then $78 \div 80(mb)$ or it will contradict either the value of the ratio σ_{el}/σ_{tot} or the value of σ_{tot} at $\sqrt{s} = 546GeV$.

The calculation gives for $\alpha = 0.04$

$$\varepsilon_1 = 0.0257 \quad n_1 = 0.35$$

and for $\alpha = 0.05$

$$\varepsilon_1 = 0.0296 \quad n_1 = 0.407.$$

Hence, as the result we have for $\alpha = 0.05$

$$\varepsilon = 0.066 + 0.03 = 0.096; \quad \varepsilon^B = 0.126 + 0.03 = 0.156;$$

$$n = 0.926 + 0.407 = 1.333; \quad n^B = 1.73 + 0.41 = 2.14.$$

The situation does not change if we consider $R^2 = R_0^2(1 + \alpha\sqrt{ln(s)})$ (see [6]). In this case, the compromising value of $\alpha = 0.4$ and for that value we find

$$\sigma_{tot}(\sqrt{s} = 546GeV) = 61.8mb; \quad \sigma_{tot}(\sqrt{s} = 1800GeV) = 80.72mb$$

. and as a result we have practically the same values

$$\varepsilon_1 = 0.022; \quad n_1 = 0.3.$$

So we can see that in the examined energy range the power of the logarithmic growth of σ_{tot} is larger than 1 but sufficiently smaller than 2. It is clear that we have the ln^2 term in the total cross section but with a small coefficient, and now we are very far from the asymptotic range. The ratio $\sigma_{el}/\sigma_{tot} = .246$ says us the same.

In our opinion, this calculation shows that the value of $\varepsilon^{QCD} = 0.15 \div 0.17$, calculated in the framework of the QCD [10] does not contradict the value of the ordinary Pomeron intercept $= 0.08$. The ε^{QCD} is to be compared with the intercept of the eikonalized Pomeron - ε^B that is equal to 0.156, as it is evident from our analysis of experimental data.

Our calculation was carried out with the gaussian form of the scattering amplitude. We think that our conclusion does not heavily depend on the definite form of the diffraction peak and we hope to confirm this in the neares future.

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Figure Captions

Fig.1 The dependence of σ_{tot} on R_0 for the different values of α.
(the long-dash lines are for $\sigma_{tot}(1800) = 80.03mb$ and $\sigma_{tot}(546) = 61.26mb$
the short-dash lines are for $R_0(i)$ such that $\sigma_{tot}(s, R_0) = \sigma_{tot}(s)_{eksp.}$)